Supplementary Material for
"Identifying Common Trend Determinants in Panel Data"

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Section S.1 contains proofs of the technical lemmas and the corollary. Section S.2 derives limiting distributions when a linear trend is present. Section S.3 provides simulation results. Section S.4 includes additional results and tables for the empirical application on crime rate determinants.

S.1 Proof of Lemmas

Proof of Lemma B1 Recall $e_t = \xi_t + \overline{x}_t^*$, where $\overline{x}_t^* = n^{-1}\sum_{i=1}^{n}(\mu_i t^{-\kappa_1} + \epsilon_{it} + \varepsilon_{it} t^{-\kappa_2}) = \overline{\mu} t^{-\kappa_1} + \overline{\epsilon}_t + \overline{\varepsilon}_t t^{-\kappa_2}$. We decompose $\widehat{\delta} - \delta$ as

$$\widehat{\delta} - \delta = \left(\sum_{t=1}^{T} \widetilde{\theta}_t \widetilde{\theta}_t'\right)^{-1} \sum_{t=1}^{T} \widetilde{\theta}_t e_t = A_{5,T}^{-1}\left(A_{1,T} + A_{2,nT} + A_{3,nT} + A_{4,nT}\right),$$

where

$$A_{1,T} = \sum_{t=1}^{T} \widetilde{\theta}_t \xi_t,\ \ A_{2,nT} = \overline{\mu}\sum_{t=1}^{T} \widetilde{\theta}_t t^{-\kappa_1},\ \ A_{3,nT} = \sum_{t=1}^{T} \widetilde{\theta}_t \overline{\epsilon}_t,\ \ A_{4,nT} = \sum_{t=1}^{T} \widetilde{\theta}_t \overline{\varepsilon}_t t^{-\kappa_2},\ \ A_{5,T} = \sum_{t=1}^{T} \widetilde{\theta}_t \widetilde{\theta}_t'.$$

When $\xi_t \sim I(0)$, under Assumptions 1 and 2 by functional limit theory and weak convergence to a stochastic integral (e.g., Ibragimov and Phillips (2008); Phillips (1988)), as $T \to \infty$

$$\frac{1}{T} A_{1,T} \rightsquigarrow \int_0^1 \widetilde{B}_\theta(r)\, dB_\xi(r).$$

Similarly, by Lemma 3.1 of Chang, Park, and Phillips (2001),

$$\frac{\sqrt{n}}{T^{(3/2)-\kappa_1}} A_{2,nT} = \sqrt{n}\overline{\mu} \cdot \frac{1}{T^{(3/2)-\kappa_1}} \sum_{t=1}^{T} \widetilde{\theta}_t t^{-\kappa_1} = O_p(1),$$

as $n,T\to\infty$, where $\overline{\mu}=O_p(n^{-1/2})$ and $\int_0^1 r^{-\kappa_1}dr=1/(1-\kappa_1)<\infty$ as we assume $\kappa_1\in(0,1/2)$. By Theorem 16 of Phillips and Moon (1999),

$$\frac{\sqrt{n}}{T}A_{3,nT}=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left\{\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\frac{\theta_t}{\sqrt{T}}\epsilon_{it}-\frac{1}{T}\sum_{t=1}^{T}\frac{\theta_t}{\sqrt{T}}\cdot\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\epsilon_{it}\right\}=O_p\left(1\right),$$

and similarly

$$\frac{\sqrt{n}}{T^{1-\kappa_2}}A_{4,nT}=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left\{\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\frac{\theta_t}{\sqrt{T}}\varepsilon_{it}\left(\frac{t}{T}\right)^{-\kappa_2}-\frac{1}{T}\sum_{t=1}^{T}\frac{\theta_t}{\sqrt{T}}\cdot\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\varepsilon_{it}\left(\frac{t}{T}\right)^{-\kappa_2}\right\}=O_p\left(1\right),$$

for $\kappa_2\in(0,1/2)$. Noting that

$$\frac{1}{T^2}A_{5,T}\rightsquigarrow\int_0^1\widetilde{B}_\theta\left(r\right)\widetilde{B}_\theta\left(r\right)'dr,$$

we therefore have

$$T(\widehat{\delta}-\delta)\rightsquigarrow\left(\int_0^1\widetilde{B}_\theta\left(r\right)\widetilde{B}_\theta\left(r\right)'dr\right)^{-1}\int_0^1\widetilde{B}_\theta\left(r\right)dB_\xi\left(r\right),$$

as $n,T\to\infty$ because it is assumed that $n/T\to\infty$. When $\xi_t\sim I(1)$, under Assumptions 1 and 3, $A_{1,T}$ dominates all other terms in the numerator, where

$$\frac{1}{T^2}A_{1,T}\rightsquigarrow\int_0^1\widetilde{B}_\theta\left(r\right)B_\xi\left(r\right)dr,$$

and hence the desired result follows. □

Proof of Lemma B2 First note that

$$S_{n,t}=\frac{1}{n}\sum_{i=1}^{n}\left(-(\widehat{\delta}-\delta)'\theta_t+\xi_t+x_{it}\right)^2,\tag{S.1}$$

where $x_{it}=\alpha_i+x_{it}^*$. When $\xi_t\sim I(1)$, $\widehat{\delta}-\delta=O_p(1)$ from Lemma B1 and the term $-(\widehat{\delta}-\delta)'\theta_t+\xi_t$ dominates x_{it} in (S.1). It follows that

$$\frac{1}{T^3}Z_{nT}\left(r\right)=\frac{1}{T^3}\sum_{t=1}^{[Tr]}\widetilde{t}\left\{\left(\xi_t-(\widehat{\delta}-\delta)'\theta_t\right)^2-\frac{1}{T}\sum_{s=1}^{T}\left(\xi_s-(\widehat{\delta}-\delta)'\theta_s\right)^2\right\}+o_p\left(1\right).$$

Note that

$$\begin{aligned}
\frac{1}{T^3}\sum_{t=1}^{[Tr]}\widetilde{t}\left(\xi_t-(\widehat{\delta}-\delta)'\theta_t\right)^2 &\rightsquigarrow \int_0^r \widetilde{s}\left(B_\xi(s)-D_\delta' B_\theta(s)\right)^2 ds \\
\frac{1}{T^3}\sum_{t=1}^{[Tr]}\widetilde{t}\cdot\frac{1}{T}\sum_{s=1}^{T}\left(\xi_s-(\widehat{\delta}-\delta)'\theta_s\right)^2 &\rightsquigarrow \int_0^r \widetilde{s}ds\int_0^1 \left(B_\xi(s)-D_\delta' B_\theta(s)\right)^2 ds,
\end{aligned}$$

where

$$D_\delta=\left(\int_0^1 \widetilde{B}_\theta(\nu)\,\widetilde{B}_\theta(\nu)'\,d\nu\right)^{-1}\int_0^1 \widetilde{B}_\theta(\nu)\,B_\xi(\nu)\,d\nu,$$

from Lemma B1. The desired result follows since

$$\begin{aligned}
& B_\xi(s)-D_\delta' B_\theta(s) \\
=\ & B_\xi(s)-\int_0^1 B_\xi(\nu)\,\widetilde{B}_\theta(\nu)'\,d\nu\left(\int_0^1 \widetilde{B}_\theta(\nu)\,\widetilde{B}_\theta(\nu)'\,d\nu\right)^{-1}B_\theta(s) \\
=\ & \omega_\xi W_1(s)-\omega_\xi\int_0^1 W_1(\nu)\,\widetilde{W}_m(\nu)'\,d\nu\Omega_\theta^{1/2}\left(\Omega_\theta^{1/2}\int_0^1 \widetilde{W}_m(\nu)\,\widetilde{W}_m(\nu)'\,d\nu\Omega_\theta^{1/2}\right)^{-1}\Omega_\theta^{1/2}W_m(s) \\
=\ & \omega_\xi V(s)^{1/2},
\end{aligned}$$

from (B.1). □

Proof of Lemma B3 From the supplementary proof given below, the leading terms of $Z_{nT}(r)$ are

$$\begin{aligned}
Z_{nT}(r) &= \sum_{t=1}^{[Tr]}\widetilde{t}\left(\xi_t^2-\frac{1}{T}\sum_{s=1}^{T}\xi_s^2\right)+\frac{1}{n}\sum_{i=1}^{n}\mu_i^2\sum_{t=1}^{[Tr]}\widetilde{t}\left\{t^{-2\kappa_1}-\frac{1}{T}\sum_{s=1}^{T}s^{-2\kappa_1}\right\} \\
&\quad+\frac{1}{n}\sum_{i=1}^{n}\sum_{t=1}^{[Tr]}\widetilde{t}\left\{\varepsilon_{it}^2 t^{-2\kappa_2}-\frac{1}{T}\sum_{s=1}^{T}\varepsilon_{is}^2 s^{-2\kappa_2}\right\}+o_p\left(\max\{T^{3/2},T^{2-2\kappa^*}\}\right) \\
&= C_{1,T}(r)+C_{2,nT}(r)+C_{3,nT}(r)+o_p\left(\max\{T^{3/2},T^{2-2\kappa^*}\}\right), \qquad \text{(S.2)}
\end{aligned}$$

where $\kappa^*=\kappa_1\wedge\kappa_2$. For $C_{1,T}(r)$, with $\mathbb{E}\xi_t^2=\sigma_\xi^2<\infty$, we have

$$\frac{1}{T^{3/2}}C_{1,T}(r) = \frac{1}{T^{3/2}}\sum_{t=1}^{[Tr]}\widetilde{t}\left\{\left(\xi_t^2-\sigma_\xi^2\right)-\frac{1}{T}\sum_{s=1}^{T}\left(\xi_s^2-\sigma_\xi^2\right)\right\}$$

$$
\begin{aligned}
&= \frac{1}{T^{3/2}} \sum_{t=1}^{[Tr]} \widetilde{t} \left(\xi_t^2 - \sigma_\xi^2\right) - \frac{1}{T^2} \sum_{t=1}^{[Tr]} \widetilde{t} \cdot \frac{1}{\sqrt{T}} \sum_{s=1}^{T} \left(\xi_s^2 - \sigma_\xi^2\right) \\
&\rightsquigarrow \int_0^r \widetilde{s} dB_{\xi\xi}(s) - \int_0^r \widetilde{s} ds B_{\xi\xi}(1) \\
&= \omega_{\xi\xi} \int_0^r \widetilde{s} d\mathcal{B}(s), \qquad \text{(S.3)}
\end{aligned}
$$

where $B_{\xi\xi}(s) = \omega_{\xi\xi} W(s)$ with standard Brownian motion $W(s)$ and standard Brownian bridge $\mathcal{B}(s) = W(s) - sW(1)$. For $C_{2,nT}(r)$, note that

$$
\frac{1}{T^{2-2\kappa_1}} \sum_{t=1}^{[Tr]} \widetilde{t} \left\{ t^{-2\kappa_1} - \frac{1}{T} \sum_{s=1}^{T} s^{-2\kappa_1} \right\} \to \int_0^r \left(s - \frac{1}{2}\right) \left(s^{-2\kappa_1} - \frac{1}{1-2\kappa_1}\right) ds = q\left(\kappa_1; r\right) < \infty,
$$

for $\kappa_1 \in (0, 1/2)$, and hence

$$
\frac{1}{T^{2-2\kappa_1}} C_{2,nT}(r) \xrightarrow{p} \sigma_\mu^2 q\left(\kappa_1; r\right),
$$

where $\mathbb{E}\mu_i^2 = \sigma_\mu^2 < \infty$. For $C_{3,nT}(r)$, as $\mathbb{E}\varepsilon_{it}^2 = \sigma_{\varepsilon,i}^2 < \infty$ for each i, note that

$$
\begin{aligned}
& \sum_{t=1}^{[Tr]} \widetilde{t} \left\{ \varepsilon_{it}^2 t^{-2\kappa_2} - \frac{1}{T} \sum_{s=1}^{T} \varepsilon_{is}^2 s^{-2\kappa_2} \right\} \\
&= \sum_{t=1}^{[Tr]} \widetilde{t} \left\{ \left(\varepsilon_{it}^2 - \sigma_{\varepsilon,i}^2\right) t^{-2\kappa_2} - \frac{1}{T} \sum_{s=1}^{T} \left(\varepsilon_{is}^2 - \sigma_{\varepsilon,i}^2\right) s^{-2\kappa_2} \right\} + \sigma_{\varepsilon,i}^2 \sum_{t=1}^{[Tr]} \widetilde{t} \left\{ t^{-2\kappa_2} - \frac{1}{T} \sum_{s=1}^{T} s^{-2\kappa_2} \right\} \\
&= O_p\left(T^{(3/2)-2\kappa_2}\right) + O_p\left(T^{2-2\kappa_2}\right),
\end{aligned}
$$

by Lemma 3.1 of Chang, Park, and Phillips (2001). It follows that

$$
\begin{aligned}
\frac{1}{T^{2-2\kappa_2}} C_{3,nT}(r) &= O_p(T^{-1/2}) + \frac{1}{n} \sum_{i=1}^{n} \sigma_{\varepsilon,i}^2 \cdot \frac{1}{T^{2-2\kappa_2}} \sum_{t=1}^{[Tr]} \widetilde{t} \left\{ t^{-2\kappa_2} - \frac{1}{T} \sum_{s=1}^{T} s^{-2\kappa_2} \right\} \\
&= \sigma_\varepsilon^2 q\left(\kappa_2; r\right) + o_p(1),
\end{aligned}
$$

where $\lim_{n\to\infty} n^{-1} \sum_{i=1}^{n} \sigma_{\varepsilon,i}^2 = \sigma_\varepsilon^2 < \infty$. Under $n/T \to \infty$, the desired results follow by combining these three terms and verifying the leading terms for each case. □

Proof of (S.2) in Lemma B3 When $\xi_t \sim I(0)$, $\widehat{\delta} - \delta = O_p(T^{-1})$ from Lemma B1 and the term $\xi_t + x_{it}$ dominates $(\widehat{\delta} - \delta)'\theta_t$ in (S.1). Therefore, the leading terms of $Z_{nT}(r)$ are given as

$$\begin{aligned}
&\frac{1}{n}\sum_{i=1}^{n}\sum_{t=1}^{[Tr]}\widetilde{t}\left\{(\xi_t+x_{it})^2-\frac{1}{T}\sum_{s=1}^{T}(\xi_s+x_{is})^2\right\} \\
=&\sum_{t=1}^{[Tr]}\widetilde{t}\left\{\xi_t^2-\frac{1}{T}\sum_{s=1}^{T}\xi_s^2\right\}+2\sum_{t=1}^{[Tr]}\widetilde{t}\left\{\xi_t\overline{x}_t-\frac{1}{T}\sum_{s=1}^{T}\xi_s\overline{x}_s\right\}+\frac{1}{n}\sum_{i=1}^{n}\sum_{t=1}^{[Tr]}\widetilde{t}\left\{x_{it}^2-\frac{1}{T}\sum_{s=1}^{T}x_{is}^2\right\} \\
=&Z_{1,T}(r)+2Z_{2,nT}(r)+Z_{3,nT}(r),
\end{aligned}$$

where $\overline{x}_t=n^{-1}\sum_{i=1}^{n}x_{it}$. The first term $Z_{1,T}(r)$ is the same as $C_{1,T}(r)$ in (S.3) in the proof of Lemma B3 and hence $Z_{1,T}(r)=O_p(T^{3/2})$.

For the second term $Z_{2,nT}(r)$, since $\overline{x}_t=\overline{\alpha}+\overline{\mu}t^{-\kappa_1}+\overline{\epsilon}_t+\overline{\varepsilon}_t t^{-\kappa_2}$,

$$\begin{aligned}
Z_{2,nT}(r) &= \sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{\xi}_t\overline{\alpha}+\sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{(\xi_t t^{-\kappa_1})}\overline{\mu}+\sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{(\xi_t\overline{\epsilon}_t)}+\sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{(\xi_t\overline{\varepsilon}_t t^{-\kappa_2})} \\
&= O_p\left(n^{-1/2}T^{3/2}+n^{-1/2}T^{3/2-\kappa_1}+n^{-1/2}T^{3/2}+n^{-1/2}T^{3/2-\kappa_2}\right).
\end{aligned}$$

The first two elements are obtained similarly as $Z_{1,T}(r)$ for

$$\begin{aligned}
\frac{n^{1/2}}{T^{3/2}}\sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{\xi}_t\overline{\alpha} &= \frac{1}{T^{3/2}}\sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{\xi}_t\cdot\sqrt{n}\overline{\alpha}=O_p(1) \\
\frac{n^{1/2}}{T^{3/2-\kappa_1}}\sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{(\xi_t t^{-\kappa_1})}\overline{\mu} &= \frac{1}{T^{3/2-\kappa_1}}\sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{(\xi_t t^{-\kappa_1})}\cdot\sqrt{n}\overline{\mu}=O_p(1),
\end{aligned}$$

where $\mathbb{E}\alpha_i=\mathbb{E}\mu_i=\mathbb{E}\xi_t=0$, $\kappa_1\in(0,1/2)$. The remaining two elements are similarly obtained from Theorem 16 of Phillips and Moon (1999) because

$$\begin{aligned}
\frac{n^{1/2}}{T^{3/2}}\sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{(\xi_t\overline{\epsilon}_t)} &= \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\frac{1}{T^{3/2}}\sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{(\xi_t\epsilon_{it})}=O_p(1) \\
\frac{n^{1/2}}{T^{3/2-\kappa_2}}\sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{(\xi_t\overline{\varepsilon}_t t^{-\kappa_2})} &= \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\frac{1}{T^{3/2-\kappa_2}}\sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{(\xi_t\varepsilon_{it}t^{-\kappa_2})}=O_p(1),
\end{aligned}$$

for $\mathbb{E}\xi_t\epsilon_{it}=\mathbb{E}\xi_t\varepsilon_{it}=0$ and $\kappa_2\in(0,1/2)$.

For the third term $Z_{3,nT}(r)$, since $x_{it}^2=\alpha_i^2+\mu_i^2t^{-2\kappa_1}+\epsilon_{it}^2+\varepsilon_{it}^2t^{-2\kappa_2}+2(\alpha_i\mu_i t^{-\kappa_1}+$

$\alpha_i\epsilon_{it} + \alpha_i\varepsilon_{it}t^{-\kappa_2} + \mu_i\epsilon_{it}t^{-\kappa_1} + \mu_i\varepsilon_{it}t^{-\kappa_1-\kappa_2} + \epsilon_{it}\varepsilon_{it}t^{-\kappa_2})$, a similar derivation yields

$$\begin{aligned} Z_{3,nT}(r) &= \sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{(t^{-2\kappa_1})} \cdot \frac{1}{n}\sum_{i=1}^{n} \mu_i^2 + \frac{1}{n}\sum_{i=1}^{n}\sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{(\epsilon_{it}^2 - \sigma_{\varepsilon,i}^2)} + \frac{1}{n}\sum_{i=1}^{n}\sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{(t^{-2\kappa_2}\varepsilon_{it}^2)} \\ &\quad + 2\sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{(t^{-\kappa_1})} \cdot \frac{1}{n}\sum_{i=1}^{n} \alpha_i\mu_i + \frac{2}{n}\sum_{i=1}^{n}\sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{\epsilon}_{it}\alpha_i + \frac{2}{n}\sum_{i=1}^{n}\sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{(t^{-\kappa_2}\varepsilon_{it})}\alpha_i \\ &\quad + \frac{2}{n}\sum_{i=1}^{n}\sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{(t^{-\kappa_1}\epsilon_{it})}\mu_i + \frac{2}{n}\sum_{i=1}^{n}\sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{(t^{-\kappa_1-\kappa_2}\varepsilon_{it})}\mu_i + \frac{2}{n}\sum_{i=1}^{n}\sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{(t^{-\kappa_2}\epsilon_{it}\varepsilon_{it})} \\ &= O_p\left(T^{2-2\kappa_1} + n^{-1/2}T^{3/2} + T^{2-2\kappa_2}\right) \\ &\quad + O_p\left(n^{-1/2}T^{2-\kappa_1} + n^{-1/2}T^{3/2} + n^{-1/2}T^{3/2-\kappa_2}\right) \\ &\quad + O_p\left(n^{-1/2}T^{3/2-\kappa_1} + n^{-1/2}T^{3/2-\kappa_1-\kappa_2} + n^{-1/2}T^{3/2-\kappa_2}\right), \end{aligned}$$

as $\kappa_1, \kappa_2 \in (0, 1/2)$, $\mathbb{E}\alpha_i\mu_i = \mathbb{E}\epsilon_{it}\varepsilon_{it} = 0$, $\mathbb{E}\mu_i^2 < \infty$, and $\lim_{n\to\infty} n^{-1}\sum_{i=1}^{n} \sigma_{\varepsilon,i}^2 < \infty$ with $\sigma_{\varepsilon,i}^2 = \mathbb{E}\varepsilon_{it}^2$. Note that the first and the third elements of $Z_{3,nT}(r)$ are respectively the same as $C_{2,nT}(r) = O_p(T^{2-2\kappa_1})$ and $C_{3,nT}(r) = O_p(T^{2-2\kappa_2})$ in the proof of Lemma B3.

The desired result follows because $n/T \to \infty$ ensures that the dominating terms are $Z_{1,T}(r) = O_p(T^{3/2})$ and the first $O_p(T^{2-2\kappa_1})$ and the third $O_p(T^{2-2\kappa_2})$ elements of $Z_{3,nT}(r)$ for any $\kappa_1, \kappa_2 \in (0, 1/2)$. $\square$

Proof of Lemma B4 Since

$$\widehat{\phi} = \frac{Z_{nT}(1)}{\sum_{t=1}^{T} (\widetilde{t})^2}$$

and $T^{-3}\sum_{t=1}^{T}(\widetilde{t})^2 \to 1/12$, the result readily follows from Lemma B2 for the case $\xi_t \sim I(1)$. When $\xi_t \sim I(0)$, note that

$$\int_0^1 \widetilde{s}d\mathcal{B}(s) = \int_0^1 \widetilde{s}dW(s) \sim \mathcal{N}\left(0, \frac{1}{12}\right), \tag{S.4}$$

as $\int_0^1 \widetilde{s}ds = 0$. The result then follows from Lemma B3 since $q(\kappa; 1) = -\kappa/\{2(1-\kappa)(1-2\kappa)\}$ and $q(1/4; 1) = -1/3$. $\square$

Proof of Lemma B5 Recall $\widehat{u}_t = \widetilde{S}_{nt} - \widetilde{\widehat{\phi} t}$. Lemmas B2 and B4-(i) yield that

$$\begin{aligned}
\frac{1}{T^3}\sum_{t=1}^{[Tr]}\widetilde{t}\widehat{u}_t &= \frac{1}{T^3}\sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{S}_{nt} - \widehat{\phi}\cdot\frac{1}{T^3}\sum_{t=1}^{[Tr]}(\widetilde{t})^2 \\
&\rightsquigarrow \omega_\xi^2\int_0^r \widetilde{s}\widetilde{V}(s)\,ds - 12\omega_\xi^2\int_0^1 \widetilde{\nu}\widetilde{V}(\nu)\,d\nu\cdot\int_0^r(\widetilde{s})^2\,ds \\
&= \omega_\xi^2\int_0^r \widetilde{s}\left\{\widetilde{V}(s) - \widetilde{s}\left(\int_0^1(\widetilde{\nu})^2\,d\nu\right)^{-1}\int_0^1\widetilde{\nu}\widetilde{V}(\nu)\,d\nu\right\}ds \\
&= \omega_\xi^2\int_0^r \widetilde{s}V^\tau(s)\,ds,
\end{aligned}$$

for $\int_0^1(\widetilde{\nu})^2\,d\nu = 1/12$ and hence

$$\begin{aligned}
T^{-6}\Psi_{nT}(b) &= \frac{1}{T^6}\sum_{t=1}^{T}\sum_{s=1}^{T}K\left(\frac{t-s}{Tb}\right)\left(\widehat{u}_t\widetilde{t}\right)\left(\widehat{u}_s\widetilde{s}\right) \\
&\rightsquigarrow \omega_\xi^4\int_0^1\int_0^1 K\left(\frac{r-s}{b}\right)\widetilde{r}\widetilde{s}V^\tau(r)\,V^\tau(s)\,drds,
\end{aligned}$$

using similar steps as in Sun (2004). □

Proof of Lemma B6 When $\kappa_1\wedge\kappa_2 > 1/4$, Lemmas B3-(i) and B4-(ii) yield

$$\begin{aligned}
\frac{1}{T^{3/2}}\sum_{t=1}^{[Tr]}\widetilde{t}\widehat{u}_t &= \frac{1}{T^{3/2}}\sum_{t=1}^{[Tr]}\widetilde{t}\widetilde{S}_{nt} - T^{3/2}\widehat{\phi}\cdot\frac{1}{T^3}\sum_{t=1}^{[Tr]}(\widetilde{t})^2 \\
&\rightsquigarrow \omega_{\xi\xi}\int_0^r\widetilde{s}d\mathcal{B}(s) - 12\omega_{\xi\xi}\int_0^1\widetilde{\nu}d\mathcal{B}(\nu)\cdot\int_0^r(\widetilde{s})^2\,ds \\
&= \omega_{\xi\xi}\int_0^r\widetilde{s}\left\{d\mathcal{B}(s) - \widetilde{s}ds\left(\int_0^1(\widetilde{\nu})^2\,d\nu\right)^{-1}\int_0^1\widetilde{\nu}d\mathcal{B}(\nu)\right\} \\
&= \omega_{\xi\xi}\int_0^r\widetilde{s}dW^\tau(s),
\end{aligned}$$

where $W^\tau(r)$ is a standard second-level Brownian bridge (e.g., MacNeill (1978)) defined as $W^\tau(r) = W(r) - rW(1) + 6r(1-r)\{(1/2)W(1) - \int_0^1 W(s)ds\}$, which is linearly $L_2[0,1]$ demeaned and detrended standard Brownian motion. Note that

$$d\mathcal{B}(s) - \widetilde{s}ds\left(\int_0^1(\widetilde{\nu})^2\,d\nu\right)^{-1}\int_0^1\widetilde{\nu}d\mathcal{B}(\nu)$$

$$
\begin{aligned}
&= dW(s) - dsW(1) - \left(s - \frac{1}{2}\right) ds 12 \int_0^1 \left(\nu - \frac{1}{2}\right) dW(\nu) \\
&= dW(s) - dsW(1) - \left(s - \frac{1}{2}\right) ds 12 \left\{ W(1) - \int_0^1 W(\nu) d\nu - \frac{1}{2} W(1) \right\} \\
&= dW(s) - dsW(1) + 6(1-2s) ds \left\{ \frac{1}{2} W(1) - \int_0^1 W(\nu) d\nu \right\} \\
&= dW^{\tau}(s),
\end{aligned}
$$

from (S.4) and by using integration by parts. Hence,

$$
\begin{aligned}
T^{-3} \Psi_{nT}(b) &= \frac{1}{T^3} \sum_{t=1}^{T} \sum_{s=1}^{T} K\left(\frac{|t-s|}{Tb}\right) \left(\widehat{u}_t \widetilde{t}\right) \left(\widehat{u}_s \widetilde{s}\right) \\
&\rightsquigarrow \omega_{\xi\xi}^2 \int_0^1 \int_0^1 K\left(\frac{t-s}{b}\right) \widetilde{r} \widetilde{s} dW^{\tau}(r) dW^{\tau}(s),
\end{aligned}
$$

similar to Kiefer and Vogelsang (2005).

When $\kappa_1 \wedge \kappa_2 = 1/4$ we consider the case $\kappa_1 = 1/4 < \kappa_2$; other cases can be obtained by the same derivation. From Lemmas B3-(ii) and B4-(ii), we have

$$
\begin{aligned}
\frac{1}{T^{3/2}} \sum_{t=1}^{[Tr]} \widetilde{t} \widehat{u}_t &\rightsquigarrow \left\{ \omega_{\xi\xi} \int_0^r \widetilde{s} d\mathcal{B}(s) + \sigma_\mu^2 q(1/4; r) \right\} - \left\{ 12 \omega_{\xi\xi} \int_0^1 \widetilde{\nu} d\mathcal{B}(\nu) - 4\sigma_\mu^2 \right\} \int_0^r (\widetilde{s})^2 ds \\
&= \omega_{\xi\xi} \int_0^r \widetilde{s} dW^{\tau}(s) + \sigma_\mu^2 \int_0^r \widetilde{s} \left(\widetilde{(s^{-1/2})} + 4\widetilde{s} \right) ds \\
&= \omega_{\xi\xi} \int_0^r \widetilde{s} \left\{ dW^{\tau}(s) + \frac{\sigma_\mu^2}{\omega_{\xi\xi}} \left(s^{-1/2} + 4s - 4 \right) ds \right\},
\end{aligned}
$$

where $q(1/4; r) = \int_0^r \widetilde{s} \widetilde{(s^{-1/2})} ds$ and $\int_0^1 \nu^{-1/2} d\nu = 2$, which yields the desired result above.

Finally, when $\kappa_1 \wedge \kappa_2 < 1/4$, we consider the case where $\kappa_1 < \kappa_2$. From Lemmas B3-(iii) and B4-(ii), we have

$$
\begin{aligned}
\frac{1}{T^{2-2\kappa_1}} \sum_{t=1}^{[Tr]} \widetilde{t} \widehat{u}_t &= \frac{1}{T^{2-2\kappa_1}} \sum_{t=1}^{[Tr]} \widetilde{t} \widetilde{S}_{nt} - T^{1+2\kappa_1} \widehat{\phi} \cdot \frac{1}{T^3} \sum_{t=1}^{[Tr]} (\widetilde{t})^2 \\
&\xrightarrow{p} \sigma_\mu^2 \int_0^r \widetilde{s} \widetilde{(s^{-2\kappa_1})} ds + \frac{6\kappa_1 \sigma_\mu^2}{(1-\kappa_1)(1-2\kappa_1)} \int_0^r (\widetilde{s})^2 ds \\
&= \sigma_\mu^2 \int_0^r \widetilde{s} \left\{ s^{-2\kappa_1} + \frac{6\kappa_1 s - (1+2\kappa_1)}{(1-\kappa_1)(1-2\kappa_1)} \right\} ds.
\end{aligned}
$$

The desired result follows in a similar way to the above. □

Proof of Corollary 1 Let $Z_{iT}(r) = \sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{\Delta}_{it}$. When $\xi_t \sim I(0)$, similar to the proof of Lemma B3 the dominant terms of $Z_{iT}(r)$ can be obtained as

$$\begin{aligned} Z_{iT}(r) &= \sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{(\xi_t + \epsilon_{it})^2} + 2\alpha_i \sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{(\xi_t + \epsilon_{it})} + \mu_i^2 \sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{(t^{-2\kappa_1})} \\ &\quad + 2\alpha_i\mu_i \sum_{t=1}^{[Tr]} \widetilde{t}\widetilde{(t^{-\kappa_1})} + o_p(\min\{T^{3/2}, T^{2-2\kappa_1}, T^{2-\kappa_1}\}) \\ &= Z_{iT,1}(r) + Z_{iT,2}(r) + Z_{iT,3}(r) + Z_{iT,4}(r) + o_p(\min\{T^{3/2}, T^{2-2\kappa_1}, T^{2-\kappa_1}\}) \\ &= O_p\left(T^{3/2}\right) + 2\alpha_i O_p\left(T^{3/2}\right) + \mu_i^2 O_p\left(T^{2-2\kappa_1}\right) + 2\alpha_i\mu_i O_p\left(T^{2-\kappa_1}\right). \qquad \text{(S.5)} \end{aligned}$$

If $\alpha_i\mu_i \neq 0$ and $|\alpha_i\mu_i| < \infty$ a.s., the dominant term of $Z_{iT}(r)$ becomes $Z_{iT,4}(r)$, from which

$$T^{1+\kappa_1}\widehat{\varphi}_i = \left(\frac{1}{12}\right)^{-1} 2\alpha_i\mu_i \int_0^1 \widetilde{\nu}\widetilde{(\nu^{-\kappa_1})}d\nu + o_p(1) = \frac{-12\kappa_1\alpha_i\mu_i}{(1-\kappa_1)(2-\kappa_1)} + o_p(1),$$

and

$$\begin{aligned} \frac{1}{T^{2-\kappa_1}} \sum_{t=1}^{[Tr]} \widetilde{t}\widehat{u}_{it} &= 2\alpha_i\mu_i \left\{ \int_0^r \widetilde{s}\widetilde{(s^{-\kappa_1})}ds - 12 \int_0^1 \widetilde{\nu}\widetilde{(\nu^{-\kappa_1})}d\nu \int_0^r (\widetilde{\nu})^2 d\nu \right\} + o_p(1) \\ &= \frac{\alpha_i\mu_i f(r, \kappa_1)}{(1-\kappa_1)(2-\kappa_1)} + o_p(1), \end{aligned}$$

where $f(r;\kappa) = \kappa(4r^3 - 6r^2 + 3r) - (2-\kappa)(r^2 - r + r^{1-\kappa}) + 2(1-\kappa)r^{2-\kappa}$. Therefore,

$$\begin{aligned} &\frac{1}{T^{2(2-\kappa_1)}} \sum_{t=1}^{T}\sum_{s=1}^{T} K\left(\frac{|t-s|}{Tb}\right)\left(\widetilde{t}\widehat{u}_{it}\right)\left(\widetilde{s}\widehat{u}_{is}\right) \\ &= \left(\frac{\alpha_i\mu_i}{(1-\kappa_1)(2-\kappa_1)}\right)^2 \int_0^1\int_0^1 K\left(\frac{r-s}{b}\right) f(r;\kappa_1) f(s;\kappa_1)\, drds + o_p(1), \end{aligned}$$

and hence

$$\mathcal{T}_{\varphi_i}(b) = \frac{T^{1+\kappa_1}\widehat{\varphi}_i}{\left\{\left(\frac{1}{T^3}\sum_{t=1}^{T}\left(\widetilde{t}\right)^2\right)^2 \frac{1}{T^{2(2-\kappa_1)}} \sum_{t=1}^{T}\sum_{s=1}^{T} K\left(\frac{|t-s|}{Tb}\right)\left(\widetilde{t}\widehat{u}_{it}\right)\left(\widetilde{s}\widehat{u}_{is}\right)\right\}^{1/2}}$$

$$= \frac{-\kappa_1 \text{sgn}\left(\alpha_i \mu_i\right)}{\left\{\int_0^1 \int_0^1 K\left(\frac{r-s}{b}\right) f\left(r;\kappa_1\right) f\left(s;\kappa_1\right) drds\right\}^{1/2}} + o_p\left(1\right), \quad \text{(S.6)}$$

where $\text{sgn}(c) = 1\{c > 0\} - 1\{c < 0\}$.

If $\alpha_i \mu_i = 0$ a.s., the dominant terms of $Z_{iT}(r)$ are different depending on the values of α_i and μ_i. First, if $\alpha_i = 0$ and $\mu_i \neq 0$ a.s., $Z_{iT,1}(r)$ and $Z_{iT,3}(r)$ terms in (S.5) are dominant for $\kappa_1, \kappa_2 \in (0, 1/2)$. In this case, conditional on μ_i, we can derive

$$\begin{cases} T^{3/2}\widehat{\varphi}_i \rightsquigarrow \mathcal{N}\left(0, 12\omega_i^2\right) & \text{if } \kappa_1 < 1/4 \\ T^{3/2}\widehat{\varphi}_i \rightsquigarrow \mathcal{N}\left(-4\mu_i^2, 12\omega_i^2\right) & \text{if } \kappa_1 = 1/4 \\ T^{1+2\kappa_1}\widehat{\varphi}_i \xrightarrow{p} \frac{-6\kappa_1}{(1-\kappa_1)(1-2\kappa_1)}\mu_i^2 & \text{if } \kappa_1 > 1/4 \end{cases},$$

similar to Lemma B4 and where ω_i^2 is the long run variance of $(\xi_t + \epsilon_{it})^2$, and the limit of $\sum_{t=1}^{[Tr]} \widetilde{t}\widehat{u}_{it}$ is obtained as in Lemma B6 by replacing $\lambda(r)$ and $\lambda^*(r)$ with $\left(4r + r^{-1/2} - 4\right)\mu_i^2$ and $c(\kappa_1; r)\mu_i^2$, respectively. From these two results, the limit of $\mathcal{T}_{\varphi_i}(b)$ is obtained as in Theorems 1 and 2, giving a form similar to that of $\mathcal{T}_\phi(b)$: $F_0(b)$, negatively-shifted $F_0(b)$, or a negative degenerating point that only depends on κ_1. Second, if $\alpha_i \neq 0$ and $\mu_i = 0$ a.s., $Z_{iT,1}(r) + Z_{iT,2}(r)$ are dominant for any $\kappa_1, \kappa_2 \in (0, 1/2)$, and we obtain $\mathcal{T}_{\varphi_i}(b) \rightsquigarrow F_0(b)$ as $T \to \infty$, whether Δ_{it} is negatively associated with t or unassociated with t. Finally, if $\alpha_i = \mu_i = 0$ a.s., $Z_{iT,1}(r)$ term is dominant for any $\kappa_1, \kappa_2 \in (0, 1/2)$, and we can derive the identical results as the second case.

When $\xi_t \sim I(1)$, the dominant terms of $Z_{iT}(r)$ are the same as those in Lemma B2, and hence the limiting distribution remains the same. □

Remark S (Choice of critical values) This corollary shows that, when $\xi_t \sim I(0)$ and $\alpha_i \mu_i = 0$ a.s., $\mathcal{T}_{\varphi_i}(b)$ converges to $F_0(b)$, negatively-shifted $F_0(b)$, or a negative point given in (23), depending on the value of κ_1. Unlike $\mathcal{T}_\phi(b)$, however, it converges to $F_0(b)$ for any $\kappa_1 \in (0, 1/2)$ if $\mu_i = 0$. Hence, $\mathcal{T}_{\varphi_i}(b)$ cannot fully distinguish the case where Δ_{it} is negatively associated with t from the case where Δ_{it} is unassociated with t. On the other hand, when $\alpha_i \mu_i > 0$ a.s., a direct calculation yields that the limit of $\mathcal{T}_{\varphi_i}(b)$ in (S.6) for any $\kappa_1 \in (0, 1/2)$ has range $[-25.91, -15.03]$ for $b = 0.1$, $[-19.50, -10.96]$ for $b = 0.2$, $[-16.37, -9.25]$ for $b = 0.3$, $[-14.57, -8.26]$ for $b = 0.4$, which are far below the degenerating point given in (23). Based on these findings and from the fact that we consider a one-sided test, we use the fist-stage test critical value $c_1 = -1.2$ for our empirical analysis, which is near the 10th percentile of $F_1(b)$, whether $\alpha_i\mu_i > 0$ or $\alpha_i\mu_i < 0$ a.s.

S.2 Limiting Distributions when Linear Trends Exist

Nonstationary variables often exhibit linear trends, which can be modeled using a random walk process with a drift. As discussed in Remark 3, we suppose the nonstationary common trend τ_t satisfies

$$\tau_t = \tau_{t-1} + c_\tau + \varepsilon_{\tau,t} = c_\tau t + \zeta_{\tau,t} \tag{S.7}$$

for some $c_\tau \neq 0$, where $\zeta_{\tau,t} = \sum_{s=1}^{t} \varepsilon_{\tau,t}$ with $\varepsilon_{\tau,t} \sim I(0)$ and $\tau_0 = 0$. θ_t is generated as either of the following:

$$\begin{cases} \text{(random walk)} & \theta_t = \theta_{t-1} + \varepsilon_{\theta,t} = \zeta_{\theta,t} \\ \text{(random walk with drift)} & \theta_t = \theta_{t-1} + c_\theta + \varepsilon_{\theta,t} = c_\theta t + \zeta_{\theta,t} \quad \text{for some } c_\theta \neq 0, \end{cases}$$

where $\zeta_{\theta,t} = \sum_{s=1}^{t} \varepsilon_{\theta,t}$ with $\varepsilon_{\theta,t} \sim I(0)$ and $\theta_0 = 0$. For simplicity, suppose $\theta_t \in \mathbb{R}^1$ and consider the following feasible cases.

Case S1: θ_t is a random walk with drift and ξ_t is stationary without trend Suppose $\theta_t = c_\theta t + \zeta_{\theta,t}$ and $\xi_t = \tau_t - \delta\theta_t = (c_\tau - \delta c_\theta)t + (\zeta_{\tau,t} - \delta\zeta_{\theta,t})$, where $(c_\tau - \delta c_\theta) = 0$ and $(\zeta_{\tau,t} - \delta\zeta_{\theta,t}) \sim I(0)$. This is the case when an appropriate θ_t was chosen so that it shares both the deterministic and stochastic trends with τ_t in (S.7). In other words, the linear combination $\tau_t - \delta\theta_t$ with δ eliminates both the linear trend and the stochastic trend, and hence the cointegration error ξ_t is a mean-zero stationary process without a linear trend. In this case, similar to the proof of Lemma B1, it can be shown that

$$T^{3/2}(\widehat{\delta} - \delta) = \frac{\sum_{t=1}^{T} \widetilde{\theta}_t e_t}{\sum_{t=1}^{T} (\widetilde{\theta}_t)^2} = \frac{T^{-3/2} \sum_{t=1}^{T} (c_\theta \widetilde{t} + \widetilde{\zeta}_{\theta,t})\xi_t + o_p(1)}{T^{-3} \sum_{t=1}^{T} (c_\theta \widetilde{t} + \widetilde{\zeta}_{\theta,t})^2} \rightsquigarrow \frac{c_\theta \omega_\xi \int_0^1 \widetilde{r} dW_1(r)}{c_\theta^2 \int_0^1 (\widetilde{r})^2 dr}.$$

For $S_{n,t}$ in (S.1), since $\widehat{\delta}-\delta = O_p(T^{-3/2})$, the term $(\widehat{\delta}-\delta)\theta_t$ is still dominated by the other terms $\xi_t + x_{it}$ as in the original case without linear trends. Therefore, all the lemmas for the case where $\xi \sim I(0)$ do not change and the limiting distribution of $\mathcal{T}_\phi(b)$ remains the same as in Theorems **1** and **2**. $\square$

Case S2: θ_t is a random walk with drift and ξ_t is a random walk Suppose $\theta_t = c_\theta t + \zeta_{\theta,t}$ and $\xi_t = \tau_t - \delta\theta_t = (c_\tau - \delta c_\theta)t + (\zeta_{\tau,t} - \delta\zeta_{\theta,t})$, where $(c_\tau - \delta c_\theta) = 0$ but $(\zeta_{\tau,t} - \delta\zeta_{\theta,t}) \sim I(1)$. This is the case when θ_t only shares the linear trend of τ_t in (S.7), but the detrended processes do not have a cointegrating relation. In other words, the linear combination $\tau_t - \delta\theta_t$ with δ eliminates the linear trend but not the stochastic trend. In this case, similar to the proof of Lemma B1, we can show that

$$T^{1/2}(\widehat{\delta}-\delta) = \frac{T^{-5/2}\sum_{t=1}^{T}(c_\theta\widetilde{t} + \widetilde{\zeta}_{\theta,t})\xi_t + o_p(1)}{T^{-3}\sum_{t=1}^{T}(c_\theta\widetilde{t} + \widetilde{\zeta}_{\theta,t})^2} \rightsquigarrow \frac{c_\theta\omega_\xi\int_0^1 \widetilde{r}W_1(r)dr}{c_\theta^2\int_0^1(\widetilde{r})^2dr} = D_{\delta,A},$$

and thus, for $S_{n,t}$ in (S.1), the term $\xi_t - (\widehat{\delta}-\delta)\theta_t$ still dominates x_{it}. It follows that

$$\begin{aligned}
&T^{-3}Z_{nT}(r)\\
&= \frac{1}{T^3}\sum_{t=1}^{[Tr]}\widetilde{t}\left\{\left(\xi_t - (\widehat{\delta}-\delta)(c_\theta t + \zeta_{\theta,t})\right)^2 - \frac{1}{T}\sum_{s=1}^{T}\left(\xi_s - (\widehat{\delta}-\delta)(c_\theta s + \zeta_{\theta,s})\right)^2\right\} + o_p(1)\\
&\rightsquigarrow \int_0^r \widetilde{s}\left(B_\xi(s) - D_{\delta,A}c_\theta s\right)^2 ds - \int_0^r \widetilde{s}ds\int_0^1\left(B_\xi(s) - D_{\delta,A}c_\theta s\right)^2 ds\\
&= \omega_\xi^2\int_0^r \widetilde{s}\widetilde{V_A}(s)\,ds,
\end{aligned}$$

where

$$\begin{aligned}
\left(B_\xi(s) - D_{\delta,A}c_\theta s\right)^2 &= \left\{B_\xi(s) - c_\theta\omega_\xi\int_0^1 W_1(\nu)\widetilde{\nu}d\nu\left(c_\theta^2\int_0^1\widetilde{\nu}^2d\nu\right)^{-1}c_\theta s\right\}^2\\
&= \omega_\xi^2\left\{W_1(s) - \int_0^1 W_1(\nu)\widetilde{\nu}d\nu\left(\int_0^1(\widetilde{\nu})^2d\nu\right)^{-1}s\right\}^2\\
&= \omega_\xi^2 V_A(s),
\end{aligned}$$

and ω_ξ^2 is the long run variance of ξ_t. Therefore, following the proofs of Lemmas B4 and B5, the limiting distribution is obtained as

$$\mathcal{T}_\phi(b) = \frac{T^{-3} Z_{nT}(1) \left(T^{-3} M_T\right)^{-1}}{\left\{\left(T^{-3} M_T\right)^{-1} T^{-6} \Psi_{nT}(b) \left(T^{-3} M_T\right)^{-1}\right\}^{1/2}} \rightsquigarrow \frac{\int_0^1 \widetilde{r} \widetilde{V_A}(r)\, dr}{\left\{\int_0^1 \int_0^1 K\left(\frac{r-s}{b}\right) \widetilde{r}\widetilde{s} V_A^\tau(r) V_A^\tau(s)\, drds\right\}^{1/2}},$$

where $M_T = \sum_{t=1}^T (\widetilde{t})^2$ and

$$V_A^\tau(r) = \widetilde{V_A}(r) - \widetilde{r}\left(\int_0^1 (\widetilde{\nu})^2 d\nu\right)^{-1} \int_0^1 \widetilde{\nu}\widetilde{V_A}(\nu)\, d\nu. \tag{S.8}$$

□

Case S3: θ_t is a random walk and ξ_t contains a linear trend Suppose $\theta_t = \zeta_{\theta,t}$ and $\xi_t = \tau_t - \delta\theta_t = c_\tau t + (\zeta_{\tau,t} - \delta\zeta_{\theta,t})$. This is the case that a θ_t that does not contain a linear trend is incorrectly chosen. Then the uncontrolled linear trend of τ_t, $c_\tau t$, dominates $\zeta_{\xi,t} = \zeta_{\tau,t} - \delta\zeta_{\theta,t}$ in the regression error ξ_t. In this case, whether $\zeta_{\xi,t}$ is $I(0)$ or $I(1)$, we can show that $\widehat{\delta} \to \infty$ because

$$T^{-1/2}(\widehat{\delta} - \delta) = \frac{T^{-5/2}\sum_{t=1}^T \widetilde{\theta}_t (c_\tau t + \zeta_{\xi,t}) + o_p(1)}{T^{-2}\sum_{t=1}^T (\widetilde{\theta}_t)^2} \rightsquigarrow \frac{c_\tau \int_0^1 r\widetilde{B}_\theta(r)\, dr}{\int_0^1 \widetilde{B}_\theta^2(r)\, dr} = D_{\delta,B},$$

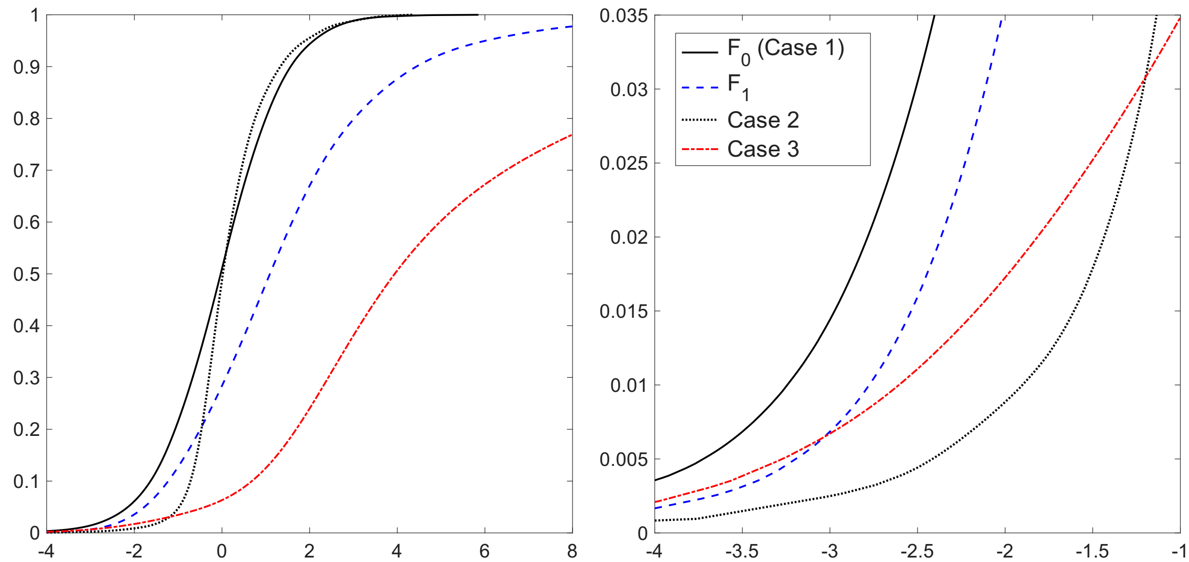
and hence, for $S_{n,t}$ in (S.1), the term $\xi_t - (\widehat{\delta} - \delta)\theta_t$ dominates x_{it}. Following the same steps as in Case S2 above,

$$\begin{aligned}
& T^{-4} Z_{nT}(r) \\
&= \frac{1}{T^4}\sum_{t=1}^{[Tr]} \widetilde{t}\left\{\left((c_\tau t + \zeta_{\xi,t}) - (\widehat{\delta} - \delta)\theta_t\right)^2 - \frac{1}{T}\sum_{s=1}^{T}\left((c_\tau s + \zeta_{\xi,s}) - (\widehat{\delta} - \delta)\theta_t\right)^2\right\} + o_p(1) \\
&\rightsquigarrow \int_0^r \widetilde{s}\left(c_\tau s - D_{\delta,B} B_\theta(s)\right)^2 ds - \int_0^r \widetilde{s} ds \int_0^1 \left(c_\tau s - D_{\delta,B} B_\theta(s)\right)^2 ds \\
&= c_\tau^2 \int_0^r \widetilde{s}\widetilde{V_B}(s)\, ds,
\end{aligned}$$

where

$$\left(c_\tau s - D_{\delta,B} B_\theta(s)\right)^2 = \left\{c_\tau s - c_\tau \int_0^1 \nu \widetilde{B}_\theta(\nu)\, d\nu \left(\int_0^1 \widetilde{B}_\theta^2(\nu)\, d\nu\right)^{-1} B_\theta(s)\right\}^2$$

Figure S1: Limiting distributions of $\mathcal{T}_\phi(b)$ with linear trends



(a) CDF's　　(b) Left-tail zoom of CDF's

Note: The figure on the left depicts the four CDF's of the limiting distributions. The black solid line is $F_0(b)$, which also applies for Case S1 when $\min\{\kappa_1, \kappa_2\} > 1/4$ as in Theorem 1; the black dotted line is for Case S2; and the red dash-dotted line is for Case S3. The blue dashed line is $F_1(b)$ without trend, as given in Figure 3. All the distribution functions are simulated with $b = 0.1$ and $T = 5,000$ using 10,000 replications. The figure on the right zooms in on the left-side tails of the CDF's.

$$
\begin{aligned}
&= c_\tau^2 \left\{ s - \omega_\theta \int_0^1 \nu \widetilde{W}_m(\nu) d\nu \left(\omega_\theta^2 \int_0^1 \widetilde{W}_m^2(\nu) d\nu \right)^{-1} \omega_\theta W_m(s) \right\}^2 \\
&= c_\tau^2 \left\{ s - \int_0^1 \nu \widetilde{W}_m(\nu) d\nu \left(\int_0^1 \widetilde{W}_m^2(\nu) d\nu \right)^{-1} W_m(s) \right\}^2 \\
&= c_\tau^2 V_B(s),
\end{aligned}
$$

and ω_θ^2 is the long run variance of θ_t. Therefore, the limiting distribution is obtained as

$$
\mathcal{T}_\phi(b) = \frac{T^{-4} Z_{nT}(1) \left(T^{-3} M_T\right)^{-1}}{\left\{ \left(T^{-3} M_T\right)^{-1} T^{-8} \Psi_{nT}(b) \left(T^{-3} M_T\right)^{-1} \right\}^{1/2}} \rightsquigarrow \frac{\int_0^1 \widetilde{r} \widetilde{V}_B(r)\, dr}{\left\{ \int_0^1 \int_0^1 K\left(\frac{r-s}{b}\right) \widetilde{r} \widetilde{s} V_B^\tau(r) V_B^\tau(s)\, dr ds \right\}^{1/2}},
$$

where $V_B^\tau(r)$ is defined as in (S.8) with $\widetilde{V}_B$. $\square$

When ξ_t is stationary without trend (hence τ_t and θ_t are cointegrated), Case S1 shows that the limiting distribution of $\mathcal{T}_\phi(b)$ remains the same as those in Theorems 1 and 2, even when τ_t and θ_t have deterministic trends. If τ_t and θ_t are not cointegrated (Case S2), or when the linear trend is not properly controlled for and hence ξ_t is trend-stationary (Case S3), the limiting distributions of $\mathcal{T}_\phi(b)$ are pivotal and behave in a similar way as the case of $\xi_t \sim I(1)$ in Theorem 1. Under the latter cases S2 and S3, $S_{n,t}$ would be positively associated with t, and the test $\mathcal{T}_\phi(b)$ should not reject $\phi \geq 0$. Figure S1 depicts the the CDF's of the limiting distributions of those cases with linear trends and their left-side tails. It shows that the the left tails of the cases S2 and S3 are much thinner than that of $F_1(b)$. Therefore, the test $\mathcal{T}_\phi(b)$ can be applied whether or not the nonstationary latent trend τ_t contain deterministic trends over the stochastic trends, using the same critical values from $F_0(b)$ as given in Tables 7 and 8.

S.3 Simulations

We use the following data generating process

$$
\begin{aligned}
y_{it} &= a_i + \tau_t + x_{it}^* \\
x_{it}^* &= \mu_i t^{-\kappa_1} + \epsilon_{it} + \varepsilon_{it} t^{-\kappa_2} \\
\tau_t &= 2\theta_t + \xi_t \\
\theta_t &= \theta_{t-1} + \varepsilon_{\theta,t},
\end{aligned} \tag{S.9}
$$

where a_i, ϵ_{it} and $\varepsilon_{\theta,t}$ are $iid\mathcal{N}(0,1)$; $\mu_i \sim iid\mathcal{N}\left(0,\sigma_\mu^2\right)$ and $\varepsilon_{it} \sim iid\mathcal{N}(0,\sigma_\varepsilon^2)$. The variances of μ_i and ε_{it} are the same $\sigma_\mu^2 = \sigma_\varepsilon^2 = \sigma^2$, and the decay rates are the same $\kappa_1 = \kappa_2 = \kappa$. When τ_t and θ_t are cointegrated, we let $\xi_t \sim iid\mathcal{N}(0,1)$; when they are not cointegrated, we let $\Delta\xi_t \sim iid\mathcal{N}(0,1)$. We simulate 5,000 times to obtain the rejection probabilities of $\mathcal{T}_\phi^0(b)$ with the critical value -1.961 for a 5% significance level from Table 1, where we use the Bartlett kernel and $b = 0.1$ in HAR estimation. The same simulations were conducted with $\mathcal{T}_\phi(b)$, giving similar results and are omitted.

Table S1 presents the rejection probabilities of the test statistic $\mathcal{T}_\phi^0(0.1)$ under the null case (i.e., Theorem 1) that $S_{n,t}$ is not negatively associated with t. The first two panels consider two scenarios: (i) the $\xi_t \sim I(0)$ case where τ_t and θ_t are cointegrated, but

Table S1: Rejection Probabilities under the Null Cases

		T				
	n	25	50	100	200	400
$F_0(b)$	25	0.052	0.056	0.045	0.041	0.047
(Case S1)	50	0.053	0.051	0.051	0.050	0.043
	100	0.052	0.052	0.049	0.049	0.045
	200	0.051	0.054	0.051	0.051	0.053
	400	0.053	0.054	0.047	0.050	0.050
$F_1(b)$	25	0.033	0.029	0.032	0.030	0.025
	50	0.032	0.030	0.028	0.027	0.021
	100	0.032	0.030	0.028	0.027	0.030
	200	0.033	0.028	0.031	0.024	0.030
	400	0.029	0.031	0.033	0.032	0.026
Case S2	25	0.018	0.008	0.006	0.006	0.005
	50	0.019	0.012	0.007	0.007	0.002
	100	0.020	0.009	0.005	0.006	0.004
	200	0.017	0.013	0.005	0.005	0.006
	400	0.015	0.008	0.006	0.007	0.004
Case S3	25	0.015	0.012	0.010	0.010	0.010
	50	0.012	0.012	0.010	0.009	0.013
	100	0.014	0.012	0.012	0.010	0.010
	200	0.011	0.013	0.012	0.013	0.010
	400	0.011	0.012	0.013	0.012	0.009

Note: '$F_0(b)$' is when $\tau_t - \delta'\theta_t = \xi_t \sim I(0)$ but x^*_{it} is not weakly $\sigma-$convergent (also for Case S1); '$F_1(b)$' is when $\xi_t \sim I(1)$; 'Case S2' is when $\xi_t \sim I(1)$ and θ_t imposes a linear trend; 'Case S3' is when ξ_t is trend-stationary.

Table S2: Rejection Probabilities under Alternative Cases

		$\kappa = 0.20$					$\kappa = 0.25$				
		T					T				
σ^2	n	25	50	100	200	400	25	50	100	200	400
1	25	0.218	0.255	0.283	0.338	0.378	0.233	0.258	0.266	0.294	0.304
	50	0.245	0.291	0.338	0.358	0.401	0.258	0.289	0.316	0.311	0.321
	100	0.269	0.301	0.338	0.403	0.445	0.285	0.298	0.315	0.346	0.352
	200	0.286	0.314	0.355	0.387	0.453	0.299	0.310	0.330	0.330	0.351
	400	0.295	0.320	0.366	0.397	0.444	0.309	0.317	0.336	0.340	0.348
3	25	0.629	0.747	0.832	0.886	0.924	0.674	0.749	0.803	0.825	0.845
	50	0.716	0.808	0.875	0.933	0.953	0.752	0.804	0.841	0.886	0.887
	100	0.761	0.856	0.921	0.943	0.974	0.785	0.846	0.887	0.895	0.921
	200	0.799	0.873	0.927	0.949	0.976	0.814	0.861	0.898	0.906	0.926
	400	0.813	0.883	0.924	0.965	0.977	0.827	0.873	0.896	0.923	0.925
5	25	0.840	0.921	0.971	0.990	0.996	0.878	0.929	0.964	0.975	0.979
	50	0.907	0.968	0.988	0.997	0.999	0.926	0.967	0.979	0.991	0.992
	100	0.944	0.978	0.993	0.998	1.000	0.951	0.975	0.989	0.993	0.996
	200	0.958	0.986	0.996	0.999	1.000	0.963	0.983	0.992	0.996	0.998
	400	0.963	0.989	0.997	0.999	1.000	0.967	0.987	0.993	0.996	0.996
10	25	0.967	0.996	1.000	1.000	1.000	0.985	0.998	1.000	1.000	1.000
	50	0.994	0.999	1.000	1.000	1.000	0.996	0.999	1.000	1.000	1.000
	100	0.996	1.000	1.000	1.000	1.000	0.997	1.000	1.000	1.000	1.000
	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

$\gamma(\xi_t^2, t)$ dominates $\gamma(R_t^x, t)$ as in (16); and (ii) the $\xi_t \sim I(1)$ case where τ_t and θ_t are not cointegrated as in (15). For these scenarios, we set $x_{it}^* = \epsilon_{it}$ with $\sigma^2 = 1$, where κ is very close to zero and hence the dominance of ξ_t's variance makes the other decaying terms irrelevant. The last two panels consider the cases with trends discussed in Section S.2: (iii) Case S2, where $\xi_t \sim I(1)$ and θ_t imposes a linear trend; and (iv) Case S3, where ξ_t is trend-stationary but θ_t does not have a linear trend. We let $c_\tau = c_\theta = 0.5$. We find that size is well controlled even in small samples. Furthermore and as discussed in Section S.2, the cases with linear trends belong to the null case.

These results are also consistent with the shapes of the limiting distributions given in Figure S1, where both $F_1(b)$ (blue dashed line) and the distribution of Case S3 (red dash-dotted line) stochastically dominate $F_0(b)$ (black solid line). While the distribution under Case S2 does not stochastically dominate $F_0(b)$, it also has a positive mode like $F_1(b)$ and a thin left tail, so that it does not affect our one-sided test.

Table S2 summarizes the rejection probabilities of $\mathcal{T}_\phi^0(0.1)$ under the alternative case (i.e., Theorem **2**) that $S_{n,t}$ is negatively associated with t. We consider $\kappa \in \{0.20, 0.25\}$ and change the values of $\sigma^2 \in \{1, 3, 5, 10\}$. As predicted from Theorem **2**, the rejection probability (i.e., power of the test) improves as σ^2 gets large, which is because the variance ratio ω_*^2 between x_{it}^* and ξ_t increases. (Recall the variance of ξ_t is fixed as unity and hence $\omega_*^2 = \sigma^2$ here.) Power improves with increasing sample sizes n, T as well, with the effect of T being more pronounced.

Next, we consider the partial convergence case by introducing 4% of non-convergent panels into the sample, which are generated as

$$y_{it} = a_i + \tau_t^{nc} + \epsilon_{it}^{nc},$$

where the idiosyncratic term $\epsilon_{it}^{nc} \sim iid\mathcal{N}(0, 4)$ does not exhibit weak $\sigma-$convergence. For the stochastic process τ_t^{nc}, we examine two specifications: one where it is cointegrated with θ_t (Table S3) and another where it is not (Table S4). In comparison, convergent panels are generated following (S.9) with $\kappa = 0.25$. From Remark S, we set the selection threshold c_1 at $\{-0.7, -1.2, -1.7\}$, corresponding approximately to the 15th, 10th, and 5th percentiles of $F_1(0.1)$, respectively.

Table S3: Subgroup Selection Probabilities (τ_t^{nc} is cointegrated with θ_t)

		$c_1 = -0.7$			$c_1 = -1.2$			$c_1 = -1.7$		
			T			T			T	
σ^2	n	50	100	200	50	100	200	50	100	200
					$\Pr(i \in \widehat{\mathcal{G}}(\theta) \mid i \notin \mathcal{G}(\theta))$					
3	50	0.276	0.265	0.254	0.169	0.142	0.141	0.083	0.072	0.086
	100	0.269	0.266	0.248	0.154	0.139	0.140	0.082	0.073	0.075
	200	0.280	0.281	0.264	0.161	0.152	0.139	0.083	0.079	0.076
10	50	0.268	0.271	0.246	0.137	0.154	0.127	0.074	0.067	0.071
	100	0.273	0.291	0.237	0.158	0.167	0.147	0.085	0.085	0.079
	200	0.276	0.266	0.260	0.152	0.154	0.127	0.079	0.078	0.067
					$\Pr(i \in \widehat{\mathcal{G}}(\theta) \mid i \in \mathcal{G}(\theta))$					
3	50	0.571	0.616	0.646	0.423	0.466	0.492	0.282	0.328	0.364
	100	0.576	0.607	0.632	0.423	0.456	0.492	0.287	0.321	0.360
	200	0.570	0.615	0.629	0.419	0.464	0.480	0.279	0.318	0.356
10	50	0.777	0.855	0.892	0.652	0.755	0.821	0.509	0.619	0.712
	100	0.777	0.851	0.892	0.650	0.752	0.815	0.501	0.618	0.698
	200	0.778	0.850	0.895	0.651	0.750	0.815	0.502	0.615	0.695
					"*Enriched*" $\Pr(i \in \widehat{\mathcal{G}}(\theta) \mid i \in \mathcal{G}(\theta))$					
3	50	0.954	0.967	0.975	0.943	0.960	0.970	0.928	0.950	0.964
	100	0.967	0.975	0.984	0.959	0.968	0.980	0.948	0.961	0.975
	200	0.974	0.983	0.987	0.967	0.978	0.985	0.958	0.972	0.982
10	50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table S4: Subgroup Selection Probabilities (τ_t^{nc} is not cointegrated with θ_t)

		$c_1=-0.7$			$c_1=-1.2$			$c_1=-1.7$		
			T			T			T	
σ^2	n	50	100	200	50	100	200	50	100	200
						$\Pr(i \in \widehat{\mathcal{G}}(\theta) \mid i \notin \mathcal{G}(\theta))$				
3	50	0.101	0.104	0.116	0.059	0.072	0.093	0.034	0.034	0.051
	100	0.128	0.101	0.095	0.088	0.068	0.058	0.049	0.037	0.031
	200	0.105	0.098	0.100	0.069	0.064	0.063	0.040	0.039	0.031
10	50	0.119	0.100	0.101	0.065	0.074	0.062	0.036	0.042	0.034
	100	0.098	0.094	0.095	0.065	0.066	0.070	0.034	0.035	0.031
	200	0.108	0.093	0.096	0.068	0.062	0.071	0.036	0.037	0.032
						$\Pr(i \in \widehat{\mathcal{G}}(\theta) \mid i \in \mathcal{G}(\theta))$				
3	50	0.610	0.679	0.725	0.471	0.543	0.606	0.329	0.409	0.474
	100	0.614	0.673	0.723	0.471	0.538	0.601	0.335	0.404	0.471
	200	0.611	0.678	0.717	0.471	0.549	0.598	0.331	0.409	0.467
10	50	0.801	0.883	0.928	0.686	0.797	0.873	0.549	0.685	0.786
	100	0.801	0.881	0.926	0.683	0.797	0.871	0.545	0.682	0.783
	200	0.801	0.881	0.925	0.687	0.797	0.868	0.547	0.680	0.784
						"*Enriched*" $\Pr(i \in \widehat{\mathcal{G}}(\theta) \mid i \in \mathcal{G}(\theta))$				
3	50	0.947	0.946	0.929	0.939	0.949	0.947	0.922	0.945	0.958
	100	0.958	0.954	0.947	0.953	0.959	0.964	0.944	0.958	0.970
	200	0.965	0.961	0.950	0.961	0.970	0.964	0.954	0.969	0.975
10	50	0.993	0.979	0.969	0.996	0.984	0.956	0.996	0.989	0.960
	100	0.994	0.980	0.967	0.997	0.985	0.968	0.998	0.990	0.973
	200	0.995	0.982	0.975	0.997	0.986	0.974	0.998	0.990	0.975

Tables S3 and S4 report the selection probabilities of both non-convergent panels, $\Pr(i \in \widehat{\mathcal{G}}(\theta)|i \notin \mathcal{G}(\theta))$, and convergent panels, $\Pr(i \in \widehat{\mathcal{G}}(\theta)|i \in \mathcal{G}(\theta))$. These results are based on 2,000 simulations using the iterative procedure detailed in Appendix A.3, with the stopping rule $|\widehat{\delta}^{(r)} - \widehat{\delta}^{(r-1)}| < 0.001$. As anticipated, the selection test becomes more conservative as c_1 decreases. The enrichment step ("*Enriched*" $\Pr(i \in \widehat{\mathcal{G}}(\theta)|i \in \mathcal{G}(\theta))$) given in Appendix A.4 ensures that convergent panels are correctly identified. The probability of correct selection increases with σ^2, reflecting the improved power of the subgroup selection test. While increasing T improves overall selection accuracy, increasing n specifically benefits the enrichment stage.

S.4 Supplementary Results for the Crime Rate Application

Table S5 provides summary statistics, data sources, and details of the data used in the crime rate applications. All values in the table are presented in levels (i.e., before log-transformation). Tables S6 and S7 report the results in Tables 3 and 6, respectively, but with the trend determinant variables for θ_t lagged by two periods, instead of one.

Table S5: Data Description and Summary Statistics

	Mean	Std. dev.	Min	Max	Period	Source
Violent Crimes						
All Violent Crime	431.29	212.88	56.85	1244.33	1987-2021	FBI UCR
Assault	277.60	143.95	34.09	785.72	1987-2021	FBI UCR
Robbery	110.20	81.50	6.40	624.66	1987-2021	FBI UCR
Homicide	5.40	3.06	0.16	20.35	1987-2021	FBI UCR
Property Crimes						
All Property Crime	3424.69	1210.91	964.70	7819.90	1987-2021	FBI UCR
Burglary	735.78	350.37	73.73	2294.26	1987-2021	FBI UCR
Larceny	2358.46	769.40	711.91	5106.13	1987-2021	FBI UCR
Motor Vehicle Theft	330.45	197.94	29.52	1157.66	1987-2021	FBI UCR
Trend Variables						
population (age 10-19)	13.92	0.10	12.76	14.69	1986-2020	Census
population (age 20-29)	14.44	0.20	13.36	17.76	1986-2020	Census
population (age 30-39)	14.77	0.27	12.87	17.06	1986-2020	Census
population (age 40-49)	13.68	0.21	10.94	15.38	1986-2020	Census
police officer	2.21	0.02	2.03	2.41	1986-2020	FBI UCR
incarceration	2.09	0.07	1.14	2.59	1986-2020	BJS
real GDP	50016.86	1312.64	36698	62606	1986-2020	FRED

Notes: (i) FBI UCR is the FBI Uniform Crime Report; Census is the U.S. Census Bureau; BJS is the Bureau of Justice Statistics; FRED is the Federal Reserve Economic Data at the St. Louis Fed. (ii) All 'Crimes' are defined as the number of crimes per 100,000 population; 'population' is the percentage of the population in each specific age group (Demog = log(population)); 'police officer' is the number of non-civilian police officers per 1,000 population (Police = log(police officer)); 'incarceration' is the incarceration count per 1,000 population (Prison = log(incarceration)); 'real GDP' is real GDP per capita in 2017 dollars (RGDP = log(real GDP)).

Table S6: Long-Run Trend Determinants for Violent Crimes (with 2-year lagged variables)

Crime	θ_t	$\hat{\delta}$	$se(\hat{\delta})$	$se^0(\hat{\delta})$	$\mathcal{T}_\phi(0.1)$	$\mathcal{T}^0_\phi(0.1)$
Violent	Demog	3.684*	0.323	0.249	-7.858*	-9.346*
	Police	-0.924	1.285	1.444	n.a.	n.a.
	Prison	-0.885*	0.208	0.214	0.380	0.541
	RGDP	-1.458*	0.142	0.132	66.116	7.435
Assault	Demog	3.127*	0.366	0.288	-4.124*	-9.247*
	Police	-0.802	1.100	1.275	n.a.	n.a.
	Prison	-0.778*	0.197	0.190	0.500	0.919
	RGDP	-1.221*	0.163	0.149	35.386	5.968
Homicide	Demog	3.124*	0.531	0.546	-6.514*	-3.538*
	Police	-2.056	1.180	1.260	n.a.	n.a.
	Prison	-0.973*	0.182	0.159	0.055	1.294
	RGDP	-1.189*	0.267	0.250	11.161	2.606
Robbery	Demog	5.780*	0.587	0.578	-20.075*	-2.582*
	Police	-0.532	2.072	2.259	n.a.	n.a.
	Prison	-1.184*	0.323	0.401	-0.668	-1.287
	RGDP	-2.367*	0.209	0.213	84.106	2.806

Table S7: Long-run Trend Determinants for Property Crimes (with 2-year lagged variables)

Crime	θ_t	$\hat{\delta}$	$se(\hat{\delta})$	$se^0(\hat{\delta})$	$\mathcal{T}_\phi(0.1)$	$\mathcal{T}^0_\phi(0.1)$	Group
Property	Demog	n.a.	n.a.	n.a.	n.a.	n.a.	0
	Police	0.280	1.500	1.760	n.a.	n.a.	n.a.
	Prison	-0.910*	0.220	0.360	-3.054*	-3.461*	40
	RGDP	n.a.	n.a.	n.a.	n.a.	n.a.	0
Burglary	Demog	3.370*	0.500	0.550	-11.708*	-12.086*	1
	Police	0.570	2.150	2.400	n.a.	n.a.	n.a.
	Prison	-1.100*	0.340	–	-3.094*	–	42
		-1.090*	–	0.500	–	-3.245*	43
	RGDP	-3.900*	0.320	–	-1.260	–	1
		n.a.	–	n.a.	–	n.a.	0
Larceny	Demog	2.080*	0.450	0.400	-1.623	-1.688	1
	Police	0.310	1.320	1.590	n.a.	n.a.	n.a.
	Prison	-0.830*	0.200	0.320	-2.948*	-3.488*	40
	RGDP	-2.270*	0.160	–	-1.238	–	1
		n.a.	–	n.a.	–	n.a.	0
Motor Vehicle Theft	Demog	5.862*	0.420	0.491	-17.238*	-2.387*	
	Police	-0.682	2.261	2.377	n.a.	n.a.	
	Prison	-1.389*	0.302	0.385	-1.097	-1.046	
	RGDP	-2.278*	0.189	0.281	53.495	1.571	

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